

Complex Numbers and Quadratic Equations

Question 1.

Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further assume that the origin, z_1 and z_2 form an equilateral triangle. Then

- (a) $a^2 = b$
- (b) $a^2 = 2b$
- (c) $a^2 = 3b$
- (d) $a^2 = 4b$

Answer: (c) $a^2 = 3b$

Given, z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$

Now, $z_1 + z_2 = -a$ and $z_1 \times z_2 = b$

Since z_1 and z_2 and z_3 form an equilateral triangle.

$$\Rightarrow z_1^2 + z_2^2 + z_3^2 = z_1 \times z_2 + z_2 \times z_3 + z_1 \times z_3$$

$$\Rightarrow z_1^2 + z_2^2 = z_1 \times z_2 \quad \{\text{since } z_3 = 0\}$$

$$\Rightarrow (z_1 + z_2)^2 - 2z_1 \times z_2 = z_1 \times z_2$$

$$\Rightarrow (z_1 + z_2)^2 = 2z_1 \times z_2 + z_1 \times z_2$$

$$\Rightarrow (z_1 + z_2)^2 = 3z_1 \times z_2$$

$$\Rightarrow (-a)^2 = 3b$$

$$\Rightarrow a^2 = 3b$$

Question 2.

The value of i^i is

- (a) 0
- (b) $e^{-\pi}$
- (c) $2e^{-\pi/2}$
- (d) $e^{-\pi/2}$

Answer: (d) $e^{-\pi/2}$

Let $A = i^i$

$$\begin{aligned} \Rightarrow \log A &= i \log i \\ \Rightarrow \log A &= i \log(0 + i) \\ \Rightarrow \log A &= i [\log 1 + i \tan^{-1} \infty] \\ \Rightarrow \log A &= i [0 + i \pi/2] \\ \Rightarrow \log A &= -\pi/2 \\ \Rightarrow A &= e^{-\pi/2} \end{aligned}$$

Question 3.

The value of $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$ is

- (a) 13 i
- (b) -13 i
- (c) 17 i
- (d) -17 i

Answer: (c) 17 i

$$\begin{aligned} \text{Given, } \sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9} \\ &= \sqrt{\{-1\} \times (25)} + 3\sqrt{\{-1\} \times 4} + 2\sqrt{\{-1\} \times 9} \\ &= \sqrt{-1} \times \sqrt{25} + 3\{\sqrt{-1} \times \sqrt{4}\} + 2\{\sqrt{-1} \times \sqrt{9}\} \\ &= 5i + 3 \times 2i + 2 \times 3i \text{ \{since } \sqrt{-1} = i\} \\ &= 5i + 6i + 6i \\ &= 17 i \\ \text{So, } \sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9} &= 17 i \end{aligned}$$

Question 4.

If the cube roots of unity are 1, ω and ω^2 , then the value of $(1 + \omega / \omega^2)^3$ is

- (a) 1
- (b) -1
- (c) ω
- (d) ω^2

Answer: (b) -1

Given, the cube roots of unity are 1, ω and ω^2

$$\text{So, } 1 + \omega + \omega^2 = 0$$

$$\text{and } \omega^3 = 1$$

$$\text{Now, } \{(1 + \omega) / \omega^2\}^3 = \{-\omega^2 / \omega^2\}^3 = \{-1\}^3 = -1$$

Question 5.

If $\{(1 + i)/(1 - i)\}^n = 1$ then the least value of n is

- (a) 1
- (b) 2

- (c) 3
(d) 4

Answer: (d) 4

Given, $\{(1+i)/(1-i)\}^n = 1$

$$\Rightarrow [\{(1+i) \times (1+i)\} / \{(1-i) \times (1+i)\}]^n = 1$$

$$\Rightarrow [\{(1+i)^2\} / \{(1-i^2)\}]^n = 1$$

$$\Rightarrow [(1+i^2+2i) / \{1-(-1)\}]^n = 1$$

$$\Rightarrow [(1-1+2i) / \{1+1\}]^n = 1$$

$$\Rightarrow [2i/2]^n = 1$$

$$\Rightarrow i^n = 1$$

Now, i^n is 1 when $n = 4$

So, the least value of n is 4

Question 6.

The value of $[i^{19} + (1/i)^{25}]^2$ is

- (a) -1
(b) -2
(c) -3
(d) -4

Answer: (d) -4

Given, $[i^{19} + (1/i)^{25}]^2$

$$= [i^{19} + 1/i^{25}]^2$$

$$= [i^{16} \times i^3 + 1/(i^{24} \times i)]^2$$

$$= [1 \times i^3 + 1/(1 \times i)]^2 \text{ \{since } i^4 = 1\}}$$

$$= [i^3 + 1/i]^2$$

$$= [i^2 \times i + 1/i]^2$$

$$= [(-1) \times i + 1/i]^2 \text{ \{since } i^2 = -1\}}$$

$$= [-i + 1/i]^2$$

$$= [-i + i^4/i]^2$$

$$= [-i + i^3]^2$$

$$= [-i + i^2 \times i]^2$$

$$= [-i + (-1) \times i]^2$$

$$= [-i - i]^2$$

$$= [-2i]^2$$

$$= 4i^2$$

$$= 4 \times (-1)$$

$$= -4$$

So, $[i^{19} + (1/i)^{25}]^2 = -4$

Question 7.

If z and w be two complex numbers such that $|z| \leq 1$, $|w| \leq 1$ and $|z + iw| = |z - iw| = 2$, then z equals { w is conjugate of w }

- (a) 1 or i
- (b) i or $-i$
- (c) 1 or -1
- (d) i or -1

Answer: (c) 1 or -1

Given $|z + iw| = |z - iw| = 2$ { w is conjugate of w }

$$\Rightarrow |z - (-iw)| = |z - (iw)| = 2$$

$$\Rightarrow |z - (-iw)| = |z - (-iw)|$$

So, z lies on the perpendicular bisector of the line joining $-iw$ and $-iw$.

Since, $-iw$ is the mirror in the x -axis, the locus of z is the x -axis.

Let $z = x + iy$ and $y = 0$

$$\Rightarrow |z| < 1 \text{ and } x^2 + 0^2 < 0$$

$$\Rightarrow -1 \leq x \leq 1$$

So, z may take value 1 or -1

Question 8.

The value of $\{-\sqrt{-1}\}^{4n+3}$, $n \in \mathbb{N}$ is

- (a) i
- (b) $-i$
- (c) 1
- (d) -1

Answer: (a) i

Given, $\{-\sqrt{-1}\}^{4n+3}$

$$= \{-i\}^{4n+3} \text{ \{since } \sqrt{-1} = i\}}$$

$$= \{-i\}^{4n} \times \{-i\}^3$$

$$= \{(-i)^4\}^n \times (-i^3) \text{ \{since } i^4 = 1\}}$$

$$= 1^n \times (-i \times i^2)$$

$$= -i \times (-1) \text{ \{since } i^2 = -1\}}$$

$$= i$$

Question 9.

Find real θ such that $(3 + 2i \times \sin \theta)/(1 - 2i \times \sin \theta)$ is real

- (a) π
- (b) $n\pi$

- (c) $n\pi/2$
 (d) $2n\pi$

Answer: (b) $n\pi$

Given,

$$(3 + 2i \times \sin \theta)/(1 - 2i \times \sin \theta) = \{(3 + 2i \times \sin \theta) \times (1 - 2i \times \sin \theta)\} / (1 - 4i^2 \times \sin^2 \theta)$$

$$(3 + 2i \times \sin \theta)/(1 - 2i \times \sin \theta) = \{(3 - 4\sin^2 \theta) + 8i \times \sin \theta\} / (1 + 4\sin^2 \theta) \dots\dots\dots 1$$

Now, equation 1 is real if $\sin \theta = 0$

$$\Rightarrow \sin \theta = \sin n\pi$$

$$\Rightarrow \theta = n\pi$$

Question 10.

If $i = \sqrt{-1}$ then $4 + 5(-1/2 + i\sqrt{3}/2)^{334} + 3(-1/2 + i\sqrt{3}/2)^{365}$ is equals to

- (a) $1 - i\sqrt{3}$
 (b) $-1 + i\sqrt{3}$
 (c) $i\sqrt{3}$
 (d) $-i\sqrt{3}$

Answer: (c) $i\sqrt{3}$

$$\text{Given, } 4 + 5(-1/2 + i\sqrt{3}/2)^{334} + 3(-1/2 + i\sqrt{3}/2)^{365}$$

$$= 4 + 5w^{334} + 3w^{365} \text{ \{since } w = -1/2 + i\sqrt{3}/2\}}$$

$$= 4 + 5w + 3w^2 \text{ \{since } w^3 = 1\}}$$

$$= 4 + 5(-1/2 + i\sqrt{3}/2) + 3(-1/2 - i\sqrt{3}/2) \text{ \{since } w^2 = (-1/2 - i\sqrt{3}/2)\}}$$

$$= i\sqrt{3}$$

Question 11.

The real part of the complex number $\sqrt{9} + \sqrt{-16}$ is

- (a) 3
 (b) -3
 (c) 4
 (d) -4

Answer: (a) 3

$$\text{Given, } \sqrt{9} + \sqrt{-16} = \sqrt{9} + \sqrt{16} \times \sqrt{-1}$$

$$= 3 + 4i \text{ \{since } i = \sqrt{-1}\}}$$

So, the real part of the complex number is 3

Question 12.

The modulus of $5 + 4i$ is

- (a) 41

- (b) -41
- (c) $\sqrt{41}$
- (d) $-\sqrt{41}$

Answer: (c) $\sqrt{41}$

Let $Z = 5 + 4i$

Now modulus of Z is calculated as

$$|Z| = \sqrt{5^2 + 4^2}$$

$$\Rightarrow |Z| = \sqrt{25 + 16}$$

$$\Rightarrow |Z| = \sqrt{41}$$

So, the modulus of $5 + 4i$ is $\sqrt{41}$

Question 13.

The modulus of $1 + i\sqrt{3}$ is

- (a) 1
- (b) 2
- (c) 3
- (d) None of these

Answer: (b) 2

Let $Z = 1 + i\sqrt{3}$

Now modulus of Z is calculated as

$$|Z| = \sqrt{1^2 + (\sqrt{3})^2}$$

$$\Rightarrow |Z| = \sqrt{1 + 3}$$

$$\Rightarrow |Z| = \sqrt{4}$$

$$\Rightarrow |Z| = 2$$

So, the modulus of $1 + i\sqrt{3}$ is 2

Question 14.

The value of $\{-\sqrt{-1}\}^{4n+3}$, $n \in \mathbb{N}$ is

- (a) i
- (b) $-i$
- (c) 1
- (d) -1

Answer: (a) i

Given, $\{-\sqrt{-1}\}^{4n+3}$

$$= \{-i\}^{4n+3} \text{ \{since } \sqrt{-1} = i\}$$

$$= \{-i\}^{4n} \times \{-i\}^3$$

$$= \{(-i)^4\}^n \times (-i^3) \text{ \{since } i^4 = 1\}}$$

$$\begin{aligned}
 &= 1^n \times (-i \times i^2) \\
 &= -i \times (-1) \text{ \{since } i^2 = -1\} \\
 &= i
 \end{aligned}$$

Question 15.

If ω is cube root of unity ($\omega \neq 1$), then the least value of n where n is a positive integer such that $(1 + \omega^2)^n = (1 + \omega^4)^n$ is

- (a) 2
- (b) 3
- (c) 5
- (d) 6

Answer: (b) 3

Given ω is an imaginary cube root of unity.

So $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

Now, $(1 + \omega^2)^n = (1 + \omega^4)^n$

$$\Rightarrow (-1)^n \times (\omega)^n = (1 + \omega \times \omega^3)^n$$

$$\Rightarrow (-1)^n \times (\omega)^n = (1 + \omega)^n$$

$$\Rightarrow (-1)^n \times (\omega)^n = (-\omega^2)^n$$

$$\Rightarrow (-1)^n \times (\omega)^n = (-1)^n \times \omega^{2n}$$

$$\Rightarrow \omega^n = \omega^{2n}$$

Since $\omega^3 = 1$, So the least value of n is 3

Question 16.

The value of $i^9 + i^{10} + i^{11} + i^{12}$ is

- (a) i
- (b) $2i$
- (c) 0
- (d) 1

Answer: (c) 0

Given, $i^9 + i^{10} + i^{11} + i^{12}$

$$= i^9 (1 + i + i^2 + i^3)$$

$$= i^9 (1 + i - 1 - i) \text{ \{since } i^2 = (-1) \text{ and } i^4 = 1\}}$$

$$= i^9 \times 0$$

$$= 0$$

Question 17.

If $a = \cos \alpha + i \sin \alpha$ and $b = \cos \beta + i \sin \beta$, then the value of $1/2(ab + 1/ab)$ is

- (a) $\sin (\alpha + \beta)$
- (b) $\cos (\alpha + \beta)$
- (c) $\sin (\alpha - \beta)$
- (d) $\cos (\alpha - \beta)$

Answer: (b) $\cos (\alpha + \beta)$

Given $a = \cos \alpha + i \sin \alpha$ and $b = \cos \beta + i \sin \beta$

Now, $1/a = 1/(\cos \alpha + i \sin \alpha)$

$$\Rightarrow 1/a = \{1 \times (\cos \alpha - i \sin \alpha) / \{(\cos \alpha + i \sin \alpha) \times (\cos \alpha + i \sin \alpha)\}$$

$$\Rightarrow 1/a = (\cos \alpha - i \sin \alpha) / (\cos^2 \alpha + i \sin^2 \alpha)$$

$$\Rightarrow 1/a = (\cos \alpha - i \sin \alpha)$$

Again, $1/b = 1/(\cos \beta + i \sin \beta)$

$$\Rightarrow 1/b = \{1 \times (\cos \beta - i \sin \beta) / \{(\cos \beta + i \sin \beta) \times (\cos \beta + i \sin \beta)\}$$

$$\Rightarrow 1/b = (\cos \beta - i \sin \beta) / (\cos^2 \beta + i \sin^2 \beta)$$

$$\Rightarrow 1/b = (\cos \beta - i \sin \beta)$$

Now, $ab = (\cos \alpha + i \sin \alpha) \times (\cos \beta + i \sin \beta)$

$$\Rightarrow ab = \cos \alpha \times \cos \beta + i \cos \alpha \times \sin \beta + i \sin \alpha \times \cos \beta - \sin \alpha \times \sin \beta$$

Again, $1/ab = (\cos \alpha - i \sin \alpha) \times (\cos \beta - i \sin \beta)$

$$\Rightarrow 1/ab = \cos \alpha \times \cos \beta - i \cos \alpha \times \sin \beta - i \sin \alpha \times \cos \beta - \sin \alpha \times \sin \beta$$

Now, $ab + 1/ab = \cos \alpha \times \cos \beta + i \cos \alpha \times \sin \beta + i \sin \alpha \times \cos \beta - \sin \alpha \times \sin \beta + \cos \alpha \times \cos \beta - i \cos \alpha \times \sin \beta - i \sin \alpha \times \cos \beta - \sin \alpha \times \sin \beta$

$$\Rightarrow ab + 1/ab = 2(\cos \alpha \times \cos \beta - \sin \alpha \times \sin \beta)$$

$$\Rightarrow 1/2(ab + 1/ab) = 2(\cos \alpha \times \cos \beta - \sin \alpha \times \sin \beta)/2$$

$$\Rightarrow 1/2(ab + 1/ab) = \cos \alpha \times \cos \beta - \sin \alpha \times \sin \beta$$

$$\Rightarrow 1/2(ab + 1/ab) = \cos(\alpha + \beta)$$

Question 18.

The polar form of $-1 + i$ is

- (a) $\sqrt{2}(\cos \pi/2 + i \times \sin \pi/2)$
- (b) $\sqrt{2}(\cos \pi/4 + i \times \sin \pi/4)$
- (c) $\sqrt{2}(\cos 3\pi/2 + i \times \sin 3\pi/2)$
- (d) $\sqrt{2}(\cos 3\pi/4 + i \times \sin 3\pi/4)$

Answer: (d) $\sqrt{2}(\cos 3\pi/4 + i \times \sin 3\pi/4)$

The polar form of a complex number $= r(\cos \theta + i \times \sin \theta)$

Given, complex number $= -1 + i$

Let $x + iy = -1 + i$

Now, $x = -1$, $y = 1$

Now, $r = \sqrt{\{(-1)^2 + 1^2\}} = \sqrt{(1 + 1)} = \sqrt{2}$

and $\tan \theta = y/x$

$$\Rightarrow \tan \theta = 1/(-1)$$

$$\Rightarrow \tan \theta = -1$$

$$\Rightarrow \theta = 3\pi/4$$

Now, polar form is $\sqrt{2}(\cos 3\pi/4 + i \times \sin 3\pi/4)$

Question 19.

For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

- (a) 0
- (b) 2
- (c) 7
- (d) 17

Answer: (b) 2

Given For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$

Now, $\text{mod}(z_1) = 12$ represents a circle centred at 0 and radius 12

$\text{mod}(z_2 - 3 - 4i) = 5$ represents a circle centred at (3, 4) and radius 5

This circle passes through the origin. Distance of diametrically opposite end is 10

So, the minimum value $(z_1 - z_2) = 2$

Question 20.

The value of $(1 - i)^2$ is

- (a) i
- (b) -i
- (c) 2i
- (d) -2i

Answer: (d) -2i

Given, $(1 - i)^2 = 1 + i^2 - 2i$

$$= 1 + (-1) - 2i$$

$$= 1 - 1 - 2i$$

$$= -2i$$
